A Cartometric Analysis of the Terrain Models of Joachim Eugen Müller (1752–1833) Using Non-contact 3D Digitizing and Visualization Techniques

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Abstract

This article assesses the accuracy of the terrain models of Joachim Eugen Müller (1752–1833) in relation to modern digital elevation data using non-contact 3D digitizing techniques. The results are objective testimony to the skill and endeavour of Joachim Eugen Müller. Using techniques primitive by modern standards, Müller provided Johann Henry Weiss (1758–1826) with data of hitherto unparalleled quality that were essential to the production of the Atlas Suisse par Meyer et Weiss. The results also demonstrate that non-contact 3D digitizing techniques not only provide a suitable data-capture method for solid terrain model analysis but are also a means of preserving digital facsimiles of such precious artefacts.

Keywords: terrain models, 3D digitizing, Joachim Eugen Müller, terrain model accuracy

1. Historical Context

The eighteenth century witnessed significant progress in methods and techniques of surveying and mapping in Europe. Relatively modern principles of surveying based on triangulation were already in use in France and Great Britain toward the end of the century. Though admirable attempts had been made to depict the high mountains of the Swiss Alps, most notably by Franz Ludwig Pfyffer (1716–1802), a sufficiently accurate map of Switzerland,
based on rigorous survey methods, did not yet exist. Only local networks of triangulation had been established, and a modern topographical survey of the whole country was a distant prospect.

This deficiency was recognized by a wealthy industrialist, Johan Rudolf Meyer (1739–1813). Meyer was an enlightened individual who, inspired by the impact of Pfyffer’s model, invested part of his fortune in funding the first systematic survey of Switzerland. Experienced at mountaineering, he was fully aware of the challenge that lay ahead, and he set about enlisting the expertise necessary to fulfill his dream. He engaged the services of the Alsatian geometer Johann Henry Weiss (1758–1826), and together they set about rehearsing their venture by ascending the Titlis peak in the summer of 1787. A carpenter from Engelberg, Joachim Eugen Müller, then aged 35, acted as a guide. Meyer discovered in Müller not only a skilful mountaineer but also an intelligent observer, particularly of topography (Imhof 1981).

By the following winter, Müller had constructed a relief model of the Engelberg area, and he began work for Meyer in the spring of 1788. Müller and Weiss then worked with the mathematician and physicist Johann George Tralles (1763–1822) of the University of Bern during the summers of 1788 and 1789. Professor Tralles, a pioneer of modern land-surveying techniques, had begun baseline surveys in different parts of Switzerland. No doubt Müller learned a significant amount about surveying principles, and particularly triangulation.

2. Survey Method

The precise survey method employed by Weiss and Müller remains unclear. Nevertheless, we know that surveying began on 10 June 1788 when Weiss, Müller, and Tralles climbed several peaks, including Hohgant, Morgenberghorn, Stockhorn, and Niesen (Wolf 1879, cited in Kloeti 1997). This early collaboration of Meyer, Weiss, Müller, and Tralles, which employed new and meticulous survey techniques, did not last, as Weiss and Meyer preferred a simpler and less time-consuming triangulation technique (Kloeti 1997).

Meyer funded the construction of a simple, but effective surveying instrument built by David Breitinger (1763–1834) of Zürich. Neither the instrument nor detailed descriptions of it have survived, but a useful description of a contemporary instrument is provided by Eduard Imhof (1981). Imhof suggests that the instrument consisted of an alidade or diopter mounted centrally on a circular disc of wood, upon which angular measurements could be plotted on a circular sheet of paper. The instrument would have been levelled using a spirit level. No angular measurements were read; a ruler attached to the alidade would have been used to simply draw a line in the direction in which the alidade was pointing. The vertical measurement was taken by rotating the sighting device on the vertical axis, the negative or positive movement of which could be read from a scale attached to the alidade in the form of a calibrated arc. The alidade did not have a sighting telescope but simply two pins at opposite ends.

Müller’s method was a form of graphic triangulation, similar in some respects to plane tabling. However, the plane table is rectangular and distinctly larger than Müller’s small disc, the map’s extent being a function of the dimensions of the plane table and the scale of survey; this enables the surveyor to record not only the azimuths of the positions of points but also their position, through intersection in the field, on the same piece of paper. Müller, by contrast, would have used a new circular sheet of paper at each new survey station and would have determined the points of intersection either at his base camp or back at his workshop.

Imhof (1981) admits that certain points remain obscure. He suggests that Müller needed a coordinate reference system to plot his points and scale as well as the orientation. A baseline measurement was also necessary to establish the coordinate system. There was no triangulated reference system at the time, so we do not know where Müller obtained his reference points. Imhof suggests that they may have been based on Müller’s preliminary work with Weiss or on reference points established by Tralles.

Though it is impossible to be certain of the precise technique employed by Müller, it would appear that he used a form of triangulation requiring the graphical transformation of his points onto a base at a pre-defined scale.
Imhof (1981) does not elaborate further on Müller’s method of plotting the intersecting points onto a base. Here we can propose theories based on the surviving paper discs. Perhaps plan errors could be adjusted by minor movements of the paper discs, which would explain the presence of slots cut along lines of measurement (see Figure 1). However, this would normally require the central hole to be the same width as the slots, allowing similar movement at the centre. Alternatively, the slots might have been cut simply to view the intersection of lines where two or more opaque paper discs overlap. Once the location of a point had been established, it would have been straightforward to mark the base underneath the discs through the coinciding holes in the overlapping discs. Indeed, this technique would have been necessary given the scale of the terrain models under construction. A disc 15 cm in diameter would create lines 7.5 cm long – 4.5 km at a scale of 1:60,000 or 9 km at 1:120,000. The slots in the disc are indeed at varying distances from the centre, and not all the lines have slots within them. Note also that the slots are marked in ink, perhaps prior to cutting, which suggests that Müller may have been able to judge the rough distance of the target from his survey station. Based on this evidence, there would appear to be little need for a sophisticated coordinate system, given the strongly graphical nature of the technique employed.

Müller would have calculated the difference in height between points using the recorded vertical angle taken from the instrument together with the horizontal distance, presumably taken from the plan plotted during the graphical triangulation process. It is unlikely that Müller could have achieved high levels of accuracy, given the low-precision instrument and technique he was using. We have no information on the vertical datum that he used and, indeed, no indication that the curvature of the Earth was taken into account. Furthermore, while we know that some 264 discs have survived (ETH Library Zurich), we do not know the total number of stations from which Müller surveyed or, indeed, the number of intersecting points he measured.

Imhof (1981) uses various assumptions to estimate a density of one station per 200 km$^2$, which he admits may be a little excessive, as this density corresponds to that of the current Swiss third-order triangulation. He suggests that the number of points obtained by intersection must be some 10 times larger than the number of survey stations. Müller probably had between 500 and 1000 points for the geometrical construction of his model; assuming a surface of 20,000 km$^2$, this gives a density of one point per 40 to 20 km$^2$.

Müller mapped the ground located between the points obtained on the basis of field observation and had a
portable compass that enabled him to determine the orientation of valleys and mountain peaks. His panoramas demonstrate great attention to detail and would have been an essential source for modelling the terrain.

3. Terrain-Modelling Method

Müller carried with him the tools required for modelling – including plaster. At his base camp, Müller constructed small relief models of the region he had just explored, which he then transported to Meyer at Aarau. While Müller conducted his own surveys and modelling, Weiss was engaged in the graphical triangulation of large expanses of Switzerland. He managed to measure altitudes for a significant portion of the country. When this work was sufficiently advanced, Weiss and Müller – or Müller practically alone, according to Rudolf Wolf (1879) – undertook the construction of a large relief model during the winter months spent at Aarau. They constructed a 1:60,000-scale model of the Swiss Alp and pre-Alp regions. Weiss concentrated on the east of the country, while Müller completed the rest. Toward the end of this project, Weiss developed the content and the drawing of the map. The alpine areas were then drawn, essentially, according to this large relief model. As Imhof (1981) points out, we have here the very rare and interesting case of a chart based on a terrain model rather than the other way around.

Unfortunately, the 1:60,000-scale relief model, which was 1.5 m wide and 4.5 m long, is no longer extant. Meyer had the model on display in his house in Aarau, where visitors could admire its hitherto unknown representation of a major part of Switzerland. It did not take long for the French ingénieurs géographes in Napoleon’s service to recognize the significance of the model (Bürgi 2007), and the Dépôt de la Guerre essentially confiscated it when it was on display in Paris, recompensing Meyer only one-fourth of the costs of the survey and the construction of the model. The model was of great military importance, as it showed a topographically intricate part of central Europe that had never before been mapped so accurately; the French army therefore wanted to prevent it from falling into enemy hands. The relief model is supposed to have been destroyed in 1922 (Bürgi 2007).

Fortunately, Müller was a prolific builder of relief models, at least 16 of which, at different scales, still exist in various locations throughout Switzerland (Mair and Grieder 2006). Among them is a model of the Bernese and Wallis Alps that is oriented NNW–SSE rather than north, approximately 50 × 70 cm in size at a scale of approximately 1:120,000. Various copies of this model have survived. It is of particular interest because it is one of the very first models made by Müller and because it was presented to the Bernese government in 1789, together with another model at 1:40,000 scale, which unfortunately has not survived (Wolf 1879). Meyer presented these two models when applying for the permission to extend the area near Lake Thun that Weiss, Tralles, and Müller had previously surveyed. Meyer also applied for permission to publish the surveyed area as part of his Atlas Suisse (Klöti 1997). Permission was granted, and in 1796 a first test sheet of the Atlas Suisse was published. This map, titled “Carte d’une partie très intéressante de la Suisse” (“Map of a very interesting part of Switzerland”), is at the 1:120,000 scale of the later Atlas Suisse and has the exactly same extension and orientation as the model of the Bernese and Wallis Alps described above. For further analyses, therefore, we are fortunate to have a relief model (Swiss Alpine Museum Model 420.00029) and a map that were both produced by Meyer’s team at about the same period and show the same geographical area.

Figure 3. Post-processing of the scans using RapidForm 2002
4. Scanning the Models

4.1 Non-contact 3D Digitizers

Non-contact 3D digitizers are used in a wide variety of application areas, including medical science (for cosmetic surgery and for fitting prostheses), in manufacturing (for reverse engineering and rapid prototyping), and for the restoration and conservation of art objects. In theory, one of the major advantages of non-contact 3D digitizers is that they operate without touching the object being digitized and therefore present minimal risk to that object.

The scanner used was a Minolta VI-900 laser scanner, is a high-resolution, high-accuracy 3D scanner that uses laser triangulation to measure distances to points. The scanner projects a near-infrared laser stripe over objects in the scene; a camera mounted in the scanner records distortions in the shape of this stripe, which is offset by a known distance from the source of the laser stripe. Minolta firmware analyses the distortions in the stripe and uses triangulation to convert the distortion to distance measurements (Piper, Ratti, and Ishii 2002). A digital image or texture of the scanned scene at 640/480 pixels is also taken by scanning the charge-couple device through an RGB filter while the stripe light is not emitted. The accuracy of the scan is moderated by adjusting the focal distance. The accuracy of the scanner using the 8 mm wide-angle lens is given as $x: \pm 1.4$ mm, $y: \pm 1.04$ mm, $z: \pm 0.64$ mm.

4.2 Set-up and Capture

The set-up of the models and the scanner is dictated by the size, physical location, and handling restrictions of the object – in our case, a solid terrain model. The scanner is mounted on a heavy-duty tripod, which allows it to be tilted sufficiently to scan a model without overhanging it, avoiding potential catastrophic damage to both scanner and model (see Figure 2). The model was therefore scanned at an oblique angle; because of the nature of relief models, this resulted in dead ground in the shadow of elevated features. Several scans from different angles were therefore required.

A wide-angle lens with a focal length of 8 mm was used to allow for an object distance range of 2 m. Because of their size, which in most cases exceeded the field of view, the models were scanned in segments. The dimensions of model 420.00029 ($74 \times 48$ cm) in the Alpine Museum, Bern, allowed it to be placed on a table; the table was then moved to enable the scanning of multiple angles and segments of the model.

The maximum recommended ambient light for the scans is 500 lx. This light level is fairly low for the subsequent capture of the colour image; images appear dark and reproduce colour poorly. Appropriate light conditions had to be achieved by moderating blinds and lighting to balance the light conditions required for the image and those required for the laser scan.

Figure 4. Digital replica of Müller’s model
5. Post-processing

Post-processing of the scans was done using RapidForm 2002. For each model, the individual scans were imported and checked. RapidForm allows the model to be viewed with texture or as a shaded relief (see Figure 3). The first step in assembling a complete model is to register the scans or “shells” to each other. The initial registration is performed by defining common points between two shells; the system then matches the overlapping areas. While performing this command, RapidForm takes into account the fact that user-selected pairs of corresponding points are not sufficiently accurate; a secondary “fine” registration automatically matches the overlapping areas and registers the shells to each other. In order to measure the success of the image registration, a shell/shell deviation measurement is calculated, which provides a colour map of the deviation and a maximum error. If the error is found to be acceptable, the two matched scans are then merged and the resulting shell used to register the next shell. This process also merges the textures of the individual shells.

The alignment of the shells in space is defined from the location of the scanner in millimetres. Because the scanner is situated at an angle to the model, the model space is tilted and not horizontal. A manual transformation of the finished shell was performed and the model moved into a horizontal model space by aligning the frame of the scanned model to a horizontal plane. The resulting shell was then exported as an XYZ text file. Unfortunately, the merged texture could not be exported satisfactorily, but it can still be viewed in RapidForm.

6. Resolution and Accuracy

The VI-900 produces a point cloud with an average distance between points of 0.44 mm. On a flat surface, the

Figure 5. (a) Swisstopo DEM (compare with Figure 5(b)). A colour version of this figure appears in Cartographica Online.
scanner produces a regular grid of points. The point density, however, is affected by the irregularity of a surface, producing increased densities on slopes that face the scanner’s laser source and having the opposite effect on slopes angled away from the scanner. Here the distances between points increase slightly to between 1 mm and 1.5 mm. The resulting point data set is therefore quite irregular, depending on the orientation of the terrain. The point density is still considered high enough for the purposes of this study, but variations in point density will have an impact on the choice of surfacing techniques for the data.

The scanner can introduce noise at distances greater than about 1.5 m. The transition is quite subtle, and because the problem was not apparent during the scanning itself, a more detailed investigation into the effects of distance on the scan results is planned for the future. In the final assembly of the model, therefore, priority was given to shells that displayed little noise; additional shells were used only to fill in dead ground from the high-quality shells. The scans of model 420.00029 did not exhibit this problem, but other, larger, models were affected. The shell/shell deviation measurements provided by RapidForm show that the maximum error in registering the shells of model 420.00029 to each other was 1.159 mm, with standard deviations between ±0.279 mm and ±0.316 mm for different shell combinations. These errors fall within the stated accuracy levels of the scanner hardware. The scanner provided a data set of considerable size and density: some 797,132 data points represent the 750 × 480 mm of the model area. On average, the distance between points was 0.44 mm, which, at a scale of 1:120,000, is equivalent to about 52 m ground distance. The final digital replica of Müller’s original model is shown in Figure 4.
7. Geo-rectifying the Model Data

In order to compare the altitudes of the Müller model with modern surveyed data, the Müller data were geo-corrected in order to make the coordinate system compatible with the modern Swiss topographic survey. The supposition here is that if we eliminate plan error as much as possible, we can then compare altitudinal differences without the extra complications of horizontal scale error, which can be looked at separately.

Geo-rectification of the Müller model was undertaken using ERDAS Imagine. Swisstopo digital elevation data re-sampled to an interval of 50 m for the same geographical area as the Müller model were available as a reference for the geo-rectification process. Control points were selected and linked for points covering the model area. The points selected were as unambiguous as possible, such as prominent peaks. Control-point residual values were examined for error, and any point errors exceeding 1 pixel contribution to the error were eliminated; 56 points remained after this process, having a root mean squared error (RMSE) of 2.23 pixels (111.5 m). An affine transformation was employed for the geo-rectification process, and the final image was converted to an XYZ ASCII format file in readiness for import into Golden Software’s Surfer software package.

8. Digital Terrain Modelling

Surfer provides sufficient modelling capabilities to provide some indication of the accuracy of Müller’s model. Gauging the level of similarity between the model and what we take as reality would require re-scaling the height values collected by the 3D Digitizer. Clearly, this could be carried out by identifying height points on the Müller model and their equivalents on the Swisstopo map. However, we decided to simply re-scale the z-values of the Müller data to the scale that was theoretically assumed to be the case after many measurements had failed to identify a consistent vertical scale. The initial vertical scale was therefore changed to 1:120,000 and Thunersee was used as a datum having an altitude of 558 m according to the Swisstopo data.

Figure 6. Rank order of principal peaks: Swisstopo, Müller, and Meyer-Weiss
Given the high density of data points, we chose triangulation with linear interpolation as the preferred interpolator. This method is an exact interpolator, with each triangle defining a plane over the grid nodes lying within the triangle. The tilt and elevation of the triangle are determined by the three original data points that define the triangle. Because the original data are used to define the triangles, the data are honoured very closely.

One of the useful capabilities of the Surfer program is its capacity to measure the difference between XYZ data points (.dat file) and a gridded surface (.grd file) for each point stored as \((x, y, z)\) values. Surfer computes the vertical difference between the \(z\)-value of the data file (e.g., Müller) and the interpolated \(z\)-value on a gridded surface (e.g., Swisstopo) using the simple formula

\[
\hat{z}_{res} = z_{dat} - z_{grd}
\]

Thus it provides a quantitative measure of how well the grid file agrees with the original data. The residual value is therefore the difference between the \(z\)-value of a point in the data file and the interpolated \(z\)-value at the same \((x, y)\) location on the gridded surface; residual values are reported as either positive (Swisstopo data lower) or negative (Swisstopo data higher).

9. Results

At this stage in our research we are interested mainly in identifying any broad trends in the differences between Müller’s model and modern surveyed points. This may throw some light on the techniques used for the survey and, indeed, for the model-making process.

Much can be learned from analysing the residuals. The pattern and amount of error (see Figure 5) demonstrate a high degree of spatial autocorrelation throughout the model. The highest errors appear to centre on the central portion of the model, the Bernese Oberland. When we consider both positive and negative residuals, we can see that the distribution of error is not random; overestimations of height are evident in the north-west and south of the model.

The central Bernese Oberland appears to be significantly underestimated. In order to examine the general trends in the error, we applied trend surface analysis to the residual values. A first-order polynomial interpolation of the residuals suggests a systematic error that follows the broad trend described above. Apart from isolated peaks in the south, the broad trend is overestimation toward the north of the model and a dip toward the south. Using the polynomial surface, we can adjust the Müller data accordingly.

The overall impression from these adjusted errors is that the Bernese Oberland, at the centre of the model, has been underestimated. A quadratic polynomial trend surface was then applied to the adjusted surface’s residuals, and another adjustment was made. The pattern of the residuals demonstrates a much lower level of spatial autocorrelation, with high residual values concentrated on valley slopes rather than on mountain peaks and valleys.

Another interesting angle to take is to compare the relative altitudes of the principal mountains with the model area (see Figure 6). This comparison is very revealing. Even over very short distances, Müller’s estimation of altitude appears to be at odds with reality. This effect is not limited to the Bernese Oberland. Interestingly, when we consider the rank order of mountain altitudes as depicted on the Meyer-Weiss Atlas de Suisse map of 1797, we find that the rank order here is in harmony with today’s data.

As with any analysis of this nature, any explanation for the distribution of error between Müller’s model and the modern survey data supplied by Swisstopo will be largely educated guesswork. Many factors could contribute to the results as outlined so far, some of them not associated with Müller’s survey and model-construction techniques. These factors include the errors introduced by the 3D digitizer and the deformation of the terrain model over time. We must also be mindful that the model may not have been mounted in its frame in its original horizontal position. Furthermore, as there is no datum identifiable on the model, an arbitrary datum had to be applied to the scanned data. Because the maximum range of height within the model is a mere 4 cm or so, these factors may well have had a significant impact on the results.

Nevertheless, even at this exploratory stage, we can establish some working hypotheses with which we can move forward in our research. First, the models appear to have been constructed by establishing principal peaks in positions that compare closely to today’s data. Intervening surfaces were perhaps modelled by “interpolation,” producing, as one would expect a lower level of accuracy in both height and plan. These principal peaks do not appear to have been modelled to the same level of accuracy in \(z\) as in \(x\) and \(y\), the emphasis being on plan accuracy rather than on height. The lower level of accuracy in the vertical axis of the model is not likely to have been due to inaccuracies in surveyed height data, as contemporary values available to Müller (as seen in the Atlas Suisse) were closer to reality.

10. Conclusions

The research described here attempted to apply scientific and objective measures in assessing the models of Joachim Eugen Müller and, of necessity, has had to forgo any appreciation of the exceptional levels of landscape modeling that Müller achieved. Throughout the paper, the term “error” appears frequently; the term is used in its statistical sense, but may still give the impression that the analysis focuses on weaknesses rather than strengths. It is
abundantly clear, however, that the models are a remarkable testimony to Müller’s dedication, skill, and artistry. Given the lack of sophisticated surveying equipment and the absence of an established triangulated survey network, his achievements are all the more remarkable. Indeed, looking at the models, it is difficult to believe that they are some 200 years old. The high standard of Müller’s work helped to establish a new benchmark in Swiss cartography and, indeed, was the progenitor of a Swiss “school” of modelling (see the excellent book by Mair and Grieder 2006).

3D non-contact digitization has proved highly suited to this type of research. It is fast, flexible, and accurate. However, we are now in a position to suggest improvements in its operation for the future. During the initial scanning process, it was not easy to gauge the success of each scan using the scanner’s built-in viewer, and problems with the scans became apparent only during post-processing. The texture capture proved especially difficult, as the necessary lighting conditions for the laser scan did not lend themselves to the capture of image information. This difficulty, combined with the relatively low resolution of $640 \times 480$, meant that the images were of low resolution and poorly exposed. A secondary image capture using better lighting and a better image sensor (e.g., a calibrated digital SLR), with subsequent image registration to the finished 3D model, would be our preferred method in future.

The results of our analysis suggest that Müller’s terrain models were made to establish the three-dimensional structure of the landscape, with an emphasis on depicting the landscape as a continually changing surface. Absolute altitudes and relative heights were perhaps of secondary concern compared to the more important problem of filling the gaps between known measured points. Müller’s talent for landscape recording ensured that the first “modern” maps of Switzerland depicted its landscape as closely as possible and ahead of the systematic triangulated surveys that began later in the nineteenth century.

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